

Electromagnetic Contributions to Vector Meson Masses and Mixings

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Abstract

We use the $1/N_c$ method to estimate electromagnetic contributions to vector meson masses and mixings. We identify several new sources of $\rho - \omega$ mixing coming from short-distance effects. We comment on the extraction of quark masses from the vector meson masses. We also present a simple group theoretical discussion of the electromagnetic mass differences.

1 Introduction

The discussion of vector meson masses and mixings has a long history. The early history can be found in the description of currents by vector meson dominance[1]. The effect of other electromagnetic effects on $\rho - \omega$ mixing was estimated to be small[2]. In this letter we will estimate these effects on $\rho - \omega$ mixing and on the individual masses. We will use the expansion in the numbers of colours (N_c) [3] as an organizing principle throughout.

The leading term is $\mathcal{O}(N_c)$ and is the mixing of ρ , ω and ϕ with the photon. This is well known and is reviewed shortly in section 3. At the next-order in $1/N_c$ or $\mathcal{O}(1)$ there also appear short distance contributions. Here we perform the long and short distance matching using the $1/N_c$ method of Buras et al.[4]. This was then extended to include proper identification of the matching scale in the calculation of the $\pi^+ - \pi^0$ electromagnetic mass difference[5]. In this method the PCAC relation to two-point functions was used. This cannot be extended beyond the chiral limit and for mesons other than the pseudoscalar octet. A method that allows this extrapolation was then applied to the $K^+ - K^0$ electromagnetic mass difference where in fact a large violation of Dashen's rule was found[6]. Variations of this method have also been used for the $K^0 - \bar{K}^0$ mass difference[7] and the kaon B_K parameter[8]. Here we apply this method to the masses of vector mesons. The long distance part of $\mathcal{O}(1)$ in the $1/N_c$ expansion we estimate simply by using cut-off photon loops. We present an argument based on heavy vector meson chiral perturbation theory to show that this is the leading contribution[9]. We perform this calculation both in the relativistic formalism and in the heavy vector meson formalism. A technical point regarding “heavy quark” integrals with an Euclidean cut-off is discussed in the appendix.

2 Effective Lagrangians and a phenomenological analysis of the vector masses

First we describe the relativistic formalism we use and then the heavy vector meson one. We will use a notation which is more appropriate for the $1/N_c$ limit than the references indicated. The various relativistic versions of describing vector mesons are all equivalent [10] and are all connected by field redefinitions. For the one connecting the antisymmetric tensor representation with the others see [11] and the others can be found in [10].

The vector mesons are collected in a matrix V_μ with

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{2}}\omega_\mu & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{2}}\omega_\mu & K_\mu^{*0} \\ K_\mu^{*-} & \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{2}}\omega_\mu & \phi_\mu \end{pmatrix}. \quad (1)$$

The pseudoscalar mesons are similarly collected in a three by three matrix M with

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}. \quad (2)$$

we do not include the η' here since that one is heavy due to the breaking of the relevant axial symmetry by the anomaly. The relativistic vector meson lagrangian we will use corresponds to model II in [10] with Lagrangian

$$\mathcal{L}_R = -\frac{1}{4}\text{tr}(D_\mu V_\nu - D_\nu V_\mu)^2 + \frac{1}{2}m_V^2\text{tr}V_\mu V^\mu + \dots. \quad (3)$$

The ellipses denote terms we do not use here. From the covariant derivative we only need the term coupling to the photon

$$D_\mu V_\nu = \partial_\mu V_\nu - ie[Q, V_\nu] + \dots, \quad (4)$$

with e the unit of charge and Q the quark charge matrix, $Q = \text{diag}(2/3, -1/3, -1/3)$.

The heavy vector meson chiral lagrangian[9] (see also [12] and [13]) is given by

$$\mathcal{L}_H = -i\text{tr}(W_\mu^\dagger v \cdot DW^\mu) + ig\text{tr}(\{W_\mu^\dagger, W_\nu\}u_\alpha)v_\beta\epsilon^{\mu\nu\alpha\beta} + a\text{tr}(\chi_+\{W_\mu^\dagger, W^\mu\}) \quad (5)$$

where we have only kept the terms leading in $1/N_c$. The matrix W_μ is the heavy meson version of V_μ and W_μ^\dagger its hermitian conjugate since now W_μ only contains the annihilation operators while W_μ^\dagger contains the creation operators. v is the velocity of the heavy meson and $D_\mu W_\nu = \partial_\mu W_\nu + [\Gamma_\mu, W_\nu]$. The matrices Γ_μ , u_μ and $f^{(+)\mu\nu}$ are defined in [10]. Here we need only

$$\Gamma_\mu = -ieQA_\mu, \quad f_{\mu\nu}^{(+)} = 2eQF_{\mu\nu} \quad \text{and} \quad u_\mu = -\frac{\sqrt{2}}{F}\partial_\mu M. \quad (6)$$

The meson mass has disappeared out of (5). Only meson mass differences appear in the heavy meson Lagrangian. In principle a term $\mu\text{tr}W_\mu^\dagger W^\mu$ exists but is removed by the choice of heavy fields. A singlet octet mass difference would be produced by a term $d\text{tr}W_\mu^\dagger\text{tr}W^\mu$ and similar terms with more traces exists for the other terms in (5). Their effects on the electromagnetic mass differences are small but we take them into account in the photon-vector-meson mixing by using the physical values of the ρ^0 , ω and ϕ masses and mixings with the photon. We also take them into account in the first phenomenological estimate given below. So there we add

$$d\text{tr}W_\mu^\dagger\text{tr}W^\mu + b\left(\text{tr}(\chi_+W_\mu^\dagger)\text{tr}(W^\mu) + \text{h.c.}\right) + c\text{tr}\chi_+\text{tr}\{W_\mu^\dagger, W^\mu\}. \quad (7)$$

The c term shifts all vector meson masses by a similar amount and will not be discussed further. The b like term is small (see below), since it is higher

order in a combined chiral and $1/N_c$ expansion, but will be kept. A term like $\text{tr} W_\mu^\dagger \text{tr}(v \cdot DW^\mu)$ treated as a $1/N_c$ correction can be removed using the equations of motion from (5).

In the limit where the quark masses vanish the leading two orders in $1/N_c$ of the electromagnetic mass difference can be described by an effective lagrangian

$$\delta_1 \text{tr} (QW_\mu^\dagger) \text{tr} (QW^\mu) + \delta_2 \text{tr} ([Q, W_\mu^\dagger] [Q, W^\mu]) + \delta_3 \text{tr} (Q^2 \{W_\mu^\dagger, W^\mu\}) . \quad (8)$$

There are no terms like $\text{tr} (Q^2 W_\mu^\dagger) \text{tr} W_\mu$ to the order we are working. Notice the difference with the pseudoscalar electromagnetic masses where only the combination $\text{tr} (QUQU^\dagger)$ appears to leading order, which is like the δ_2 term.

The meson masses and mixings obtained from these lagrangians are :

$$\begin{aligned} \Delta\rho^+ &= -\delta_2 + \frac{5}{9}\delta_3 + 4Ba(m_u + m_d) + 8Bc(m_u + m_d + m_s) \\ \Delta\rho^0 &= \frac{\delta_1}{2} + \frac{5}{9}\delta_3 + 4Ba(m_u + m_d) + 8Bc(m_u + m_d + m_s) \\ \Delta\rho\omega &= \frac{\delta_1}{6} + \frac{\delta_3}{3} + 4Ba(m_u - m_d) + 4Bb(m_u - m_d) \\ \Delta\rho\phi &= -\frac{\delta_1}{3\sqrt{2}} + 2\sqrt{2}Bb(m_u - m_d) \\ \Delta\omega &= 2d + \frac{\delta_1}{18} + \frac{5}{9}\delta_3 + 4Ba(m_u + m_d) + 8Bb(m_u + m_d) \\ &\quad + 8Bc(m_u + m_d + m_s) \\ \Delta\omega\phi &= \sqrt{2}d - \frac{\delta_1}{9\sqrt{2}} + 2\sqrt{2}Bb(m_u + m_d + 2m_s) \\ \Delta\phi &= d + \frac{\delta_1}{9} + \frac{2}{9}\delta_3 + 8Bam_s + 8Bbm_s + 8Bc(m_u + m_d + m_s) \\ \Delta K^{*+} &= -\delta_2 + \frac{5}{9}\delta_3 + 4Ba(m_s + m_u) + 8Bc(m_u + m_d + m_s) \\ \Delta K^{*0} &= \frac{2}{9}\delta_3 + 4Ba(m_s + m_d) + 8Bc(m_u + m_d + m_s) \end{aligned} \quad (9)$$

The relations

$$\begin{aligned} 0 &= m_\phi + m_{\rho^+} - m_{K^{*+}} - m_{K^{*0}} - \sqrt{2}\Delta\omega\phi + \frac{m_\omega - m_{\rho^0}}{2} \\ 0 &= \Delta\rho\omega - \sqrt{2}\Delta\rho\phi - m_{K^{*+}} + m_{K^{*0}} - m_{\rho^0} + m_{\rho^+} \end{aligned} \quad (10)$$

can easily be obtained.

We can now try to determine all these parameters from experiment. Using [14]

$$|\Delta\rho\omega|^2 = \left[(m_\rho - m_\omega)^2 + \frac{1}{4}(\Gamma_\rho - \Gamma_\omega)^2 \right] \frac{\Gamma(\omega \rightarrow \pi\pi)}{\Gamma(\rho \rightarrow \pi\pi)} \quad (11)$$

and the branching ratio of $\omega \rightarrow \pi\pi$ of $2.11 \pm 0.30\%$ [15] we obtain

$$\Delta\rho\omega = -2.5 \text{ MeV}. \quad (12)$$

where the sign is determined from the interference pattern of ρ^0 and ω in $e^+e^- \rightarrow \pi^+\pi^-$. Similar estimates can be made of the $\omega - \phi$ and the $\rho - \phi$ mixing using the branching ratios in 3π and 2π respectively. Here there can be very large corrections from Kaon rescattering into these final states so these numbers should not be regarded as very informative. We obtain

$$|\Delta\rho\phi| = 0.33 \pm 0.1 \text{ MeV} \quad \text{and} \quad |\Delta\omega\phi| = 8.1 \text{ MeV}. \quad (13)$$

We can also estimate the $\omega\phi$ mixing from the decays $\omega, \phi \rightarrow \pi^0\gamma$. We find

$$|\Delta\omega\phi| = 14.1 \text{ MeV}. \quad (14)$$

Using these numbers and the measured masses we see that the first relation is well satisfied but the second one is not. This already indicates that there will be higher order corrections.

A small technical note. We calculate here the inverse of the vector meson propagator and determine its zero. In the relativistic case we take only the $g_{\mu\nu}$ part of this inverse propagator for simplicity:

$$ig_{\mu\nu} \left(-p^2 + M^2 + \Delta(p^2) \right) = 0. \quad (15)$$

To the order we are working we now have

$$\Delta M^2 = \Delta(p^2 = M^2). \quad (16)$$

Similarly in the heavy meson case,

$$i \left(-v \cdot p + \Delta M(p^2, v \cdot p) \right) = 0. \quad (17)$$

Here the mass shift is given by

$$\Delta M = \Delta M(0, 0) = \frac{\Delta M^2}{2M}. \quad (18)$$

3 Long distance : Mixing with the photon

We can add to the Lagrangian (5) a term describing the mixing with the photon. This term has the form

$$\frac{if_V\sqrt{m_V}}{2} \text{tr} \left([e^{-im_V v \cdot x} W_\nu^\dagger - e^{im_V v \cdot x} W_\nu] v_\mu f^{(+)\mu\nu} \right) \quad (19)$$

The factor $\sqrt{m_V}$ is present because of the normalization of the W_μ field. This can be obtained from the relativistic Lagrangian

$$-\frac{f_V}{2\sqrt{2}}\text{tr}\left((D_\mu V_\nu - D_\nu V_\mu)f^{(+)\mu\nu}\right), \quad (20)$$

by inserting the relation between W_μ and V_μ , and keeping only leading terms in $1/m_V$. The parameter f_V can be estimated from the decays into e^+e^- using

$$\Gamma\left[(\rho^0, \omega, \phi) \rightarrow e^+e^-\right] = \left(1, \frac{1}{9}, \frac{2}{9}\right) \frac{4\pi\alpha^2}{3} m_V f_V^2. \quad (21)$$

This leads to $|f_V| = (0.2, 0.18, 0.16)$ MeV, showing good agreement between the three observed decays. The diagram in Fig.2.e leads to a mass difference of the form described by the δ_1 term and is given by

$$\delta_1 = 4\pi\alpha f_V^2 m_V. \quad (22)$$

If we used the relativistic formulation we would have obtained the same mass differences but with m_V replaced by the relevant vector mass. This is one possible estimate of $\mathcal{O}(m_q)$ corrections.

4 Long distance : Photon loops

Here we only have to compute the diagrams of Fig.2.a and Fig.2.b. The calculation is by rotating the integrals in Euclidean space and then cutting off on the off-shellness of the photon momentum. In the relativistic case both diagrams contribute and we obtain

$$\begin{aligned} \delta m_V^2 = & -\frac{i\alpha}{4\pi^3} \int d^4k \left\{ \left(\frac{1}{k^2 + 2p \cdot k} \right) \left(\frac{1}{3} - \frac{k^2}{3m_V^2} + 4\frac{m_V^2}{k^2} \right) + \frac{1}{3m_V^2} + \frac{1}{k^2} \right\} \\ & + 3\frac{i\alpha}{4\pi^3} \int d^4k \frac{1}{k^2} \end{aligned} \quad (23)$$

for the case of the charged ρ and K^* and zero for the others. The first line of (23) is the contribution of Fig.2.a, and the second line is the contributions of the tadpole, Fig.2.b. We have checked that the gauge dependence of the photon propagator cancels. We can evaluate all the relevant integrals and obtain

$$\begin{aligned} \delta^{LD} m_V^2 = & \frac{\alpha m_V^2}{\pi} \left\{ \frac{1}{16} \frac{\Lambda^4}{m_V^4} + \frac{1}{72} \frac{\Lambda^6}{m_V^6} + \frac{11}{6} \log \frac{\Lambda(1 + \sqrt{1 + 4m_V^2/\Lambda^2})}{2m_V} \right. \\ & \left. - \frac{\Lambda^2}{3m_V^2} \sqrt{1 + 4m_V^2/\Lambda^2} \left(-\frac{13}{8} + \frac{5}{48} \frac{\Lambda^2}{m_V^2} + \frac{1}{24} \frac{\Lambda^4}{m_V^4} \right) \right\} \end{aligned} \quad (24)$$

This is a long distance calculation so we have Λ below about 1 GeV. We can also take the approximation of Λ small and obtain

$$\Delta M^2 = \frac{\alpha}{\pi} (2m_V \Lambda + O(\Lambda^2)) \quad (25)$$

Both expressions are plotted in Fig.1 for Λ between 500 and 800 MeV.

Alternatively, we could directly have used the heavy meson lagrangian (5). Then only the diagram in Fig.2.c exists and we obtain

$$\Delta M = \frac{\alpha \Lambda}{\pi} \quad (26)$$

The relevant integral is evaluated in the appendix with an euclidean cutoff and we see that it directly reproduces the small Λ result of the relativistic version. We can also see here that in order to obtain contributions that match the short distance of the neutral mesons we have to go beyond the lowest order in the chiral expansion. This is similar to what was observed in [6] where the lowest order electromagnetic loop also gave no mass to the neutral particles where as the short distance one did.

This contribution can be summarized as

$$\delta_2^{LD} = -\frac{\alpha}{\pi} \Lambda \quad (27)$$

That it is the leading term in the combined chiral and $1/N_c$ expansions can be seen from the Lagrangian (5). Loops with pseudoscalar mesons are higher order in the chiral expansion.

5 Short distance contribution

Here we take the short distance effective action as evaluated within this approach in [6]. There were three terms there. The first one describes the electromagnetic shifts in the value of the underlying quark masses and it's effect can be absorbed in them. We therefore do not consider it any more. The Box and the Penguin-like contributions are given by

$$S_{eff}^2 = \frac{\alpha \alpha_S}{27\Lambda^2} \left\{ 4\bar{u}u + \bar{d}d + \bar{s}s \right\}_{V_{\alpha\beta}} \left\{ \bar{u}u + \bar{d}d + \bar{s}s \right\}_{V_{\beta\alpha}} \quad (28)$$

$$S_{eff}^3 = \frac{-\alpha \alpha_S}{6\Lambda^2} \left\{ -4(uu) - (dd) - (ss) + 4(ud) + 4(us) - 2(ds) \right\} \quad (29)$$

up to terms of order m_q^2 . Here $\{\bar{q}q\}_{V_{\alpha\beta}} = \bar{q}_\alpha \gamma_\mu q_\beta$ and $(qq') = (\bar{q}_\alpha \gamma_\mu \gamma_5 q_\beta)(\bar{q}'_\beta \gamma_\mu \gamma_5 q'_\alpha)$. It now remains to estimate the matrix elements of these operators between vector meson states. In leading order in $1/N_c$ we can Fierz the operators in (29). The matrix elements are then in leading $1/N_c$

$$\begin{aligned} \langle \rho_1^0 | \bar{u}_\alpha(\gamma_5) \gamma_\mu u_\beta \bar{u}_\beta(\gamma_5) \gamma^\mu u_\alpha | \rho_2^0 \rangle &= -(+)2 \langle \bar{u}u \rangle \langle \rho_1^0 | \bar{u}u | \rho_2^0 \rangle + \langle \rho_1^0 | \bar{u} \gamma_\mu u | 0 \rangle \langle 0 | \bar{u} \gamma^\mu u | \rho_2^0 \rangle \\ &= \left(-(+)8B_0^2 F^2 B_0 a + \frac{1}{2} f_V^2 m_V^3 \right) \epsilon_1 \cdot \epsilon_2. \end{aligned} \quad (30)$$

This can then be evaluated from (5) since the relevant couplings to external currents are there and leads to the second line in (30). All other matrix elements can be obtained in a similar way. The relevant formulae in the chiral limit for δ_2^{SD} and δ_3^{SD} are

$$\begin{aligned}\delta_2^{SD} &= \frac{3\alpha\alpha_S}{4\Lambda^2} f_V^2 m_V^3 \\ \delta_3^{SD} &= \frac{\alpha\alpha_S}{12\Lambda^2} [11f_V^2 m_V^3 + 112B_0^2 F^2 a],\end{aligned}\tag{31}$$

and sum of the short and long distance contributions read

$$\begin{aligned}\delta_2 &= -\frac{\alpha}{\pi}\Lambda + \frac{3\alpha\alpha_s}{4\Lambda^2} f_V^2 m_V^3 \\ \delta_3 &= \frac{\alpha\alpha_s}{12\Lambda^2} [11f_V^2 m_V^3 + 112B_0^2 F^2 a]\end{aligned}\tag{32}$$

We cannot simply estimate the corrections due to the quark masses here as was done for the pseudoscalars in [6]. The reason is that we would need information about the terms with 2 powers of the quark masses for the vector meson masses. These are probably rather small since the linear fit from section 2 works rather well. This means that we cannot estimate the quark breaking effects in the terms proportional to $B_0 a$. We can however estimate them in the $f_V^2 m_V^3$ terms by taking the measured values for these. This will be done in section 7

6 Long distance : Meson loops

In this section we temporarily leave the framework of the $1/N_c$ expansion and check whether there are any well defined contributions, nonanalytic in the quark masses that might contribute. The mass differences of vector mesons have contributions of order $m_q^{3/2}$ [9]. So the electromagnetic part of the mass splittings for the pseudoscalars in fact induces a $e^2\sqrt{m_q}$ type of correction which is the leading quark mass correction to the vector meson electromagnetic masses. These come from the diagrams like Fig.2.d. We find

$$\begin{aligned}\Delta\rho^+ &= -\frac{g^2}{6\pi F_\pi^2} \left\{ m_{K^+}^3 + m_{K^0}^3 + 2m_{\pi^+}^3 + \frac{2}{3}m_\eta^3 \right\} \\ \Delta\rho^0 &= -\frac{g^2}{6\pi F_\pi^2} \left\{ m_{K^+}^3 + m_{K^0}^3 + 2m_{\pi^0}^3 + \frac{2}{3}m_\eta^3 \right\} \\ \Delta\rho\omega &= -\frac{g^2}{6\pi F_\pi^2} \left\{ m_{K^+}^3 - m_{K^0}^3 \right\} \\ \Delta\rho\phi &= -\frac{\sqrt{2}g^2}{6\pi F_\pi^2} \left\{ m_{K^+}^3 - m_{K^0}^3 \right\}\end{aligned}$$

$$\begin{aligned}
\Delta\omega &= -\frac{g^2}{6\pi F_\pi^2} \left\{ m_{K^+}^3 + m_{K^0}^3 + \frac{2}{3}m_\eta^3 + 4m_{\pi^+}^3 + 2m_{\pi^0}^3 \right\} \\
\Delta\omega\phi &= -\frac{\sqrt{2}g^2}{6\pi F_\pi^2} \left\{ m_{K^+}^3 + m_{K^0}^3 \right\} \\
\Delta\phi &= -\frac{g^2}{6\pi F_\pi^2} \left\{ 2m_{K^+}^3 + 2m_{K^0}^3 + \frac{8}{3}m_\eta^3 \right\} \\
\Delta K^{*+} &= -\frac{g^2}{6\pi F_\pi^2} \left\{ \frac{1}{6}m_\eta^3 + \frac{1}{2}m_{\pi^0}^3 + m_{\pi^+}^3 + 2m_{K^+}^3 + m_{K^0}^3 \right\} \\
\Delta K^{*0} &= -\frac{g^2}{6\pi F_\pi^2} \left\{ \frac{1}{6}m_\eta^3 + \frac{1}{2}m_{\pi^0}^3 + m_{\pi^+}^3 + 2m_{K^0}^3 + m_{K^+}^3 \right\} \tag{33}
\end{aligned}$$

The electromagnetic corrections are

$$\begin{aligned}
\Delta^{EM}\rho^+ &= -\frac{g^2}{4\pi F_\pi^2} \{m_{K^+} + 2m_{\pi^+}\} \delta m_{P^+}^2 = -0.92 \text{ MeV} \\
\Delta^{EM}\rho^0 &= -\frac{g^2}{4\pi F_\pi^2} m_{K^+} \delta m_{P^+}^2 = -0.59 \text{ MeV} \\
\Delta^{EM}\rho\omega &= -\frac{g^2}{4\pi F_\pi^2} m_{K^+} \delta m_{P^+}^2 = -0.59 \text{ MeV} \\
\Delta^{EM}\rho\phi &= -\frac{\sqrt{2}g^2}{4\pi F_\pi^2} m_{K^+} \delta m_{P^+}^2 = -0.83 \text{ MeV} \\
\Delta^{EM}\omega &= -\frac{g^2}{4\pi F_\pi^2} \{m_{K^+} + 4m_{\pi^+}\} \delta m_{P^+}^2 = -1.2 \text{ MeV} \\
\Delta^{EM}\omega\phi &= -\frac{\sqrt{2}g^2}{4\pi F_\pi^2} m_{K^+} \delta m_{P^+}^2 = -0.83 \text{ MeV} \\
\Delta^{EM}\phi &= -\frac{g^2}{2\pi F_\pi^2} m_{K^+} \delta m_{P^+}^2 = -1.2 \text{ MeV} \\
\Delta^{EM}K^{*+} &= -\frac{g^2}{4\pi F_\pi^2} \{m_{\pi^+} + 2m_{K^+}\} \delta m_{P^+}^2 = -1.3 \text{ MeV} \\
\Delta^{EM}K^{*0} &= -\frac{g^2}{4\pi F_\pi^2} \{m_{\pi^+} + m_{K^+}\} \delta m_{P^+}^2 = -0.75 \text{ MeV} \tag{34}
\end{aligned}$$

Here, $F_\pi = 93 \text{ MeV}$ and $\delta m_{P^+}^2 = m_{\pi^+}^2 - m_{\pi^0}^2$. The numerical value $g = 0.32$ can be obtained comparing (5) with $\mathcal{L}(\rho\rho\pi)$, (eq.(123) of [16], where VMD for the $\rho\pi\gamma$ coupling is assumed). In [9], the values $g = 0.375$ ([17]) and $g = 0.5$ are suggested coming from a quark meson model. The numbers from (33) and (34) should not be taken too seriously since some of the total corrections, (33), of order $O(m_q^{3/2})$, are huge: for the ρ , for example, we find $\delta\rho = -223 \text{ MeV}$. In addition if terms of order $m_P\Delta m_V$, which are formally higher order, in the expansion of the integrals are kept there are large numerical cancellations. At

that order there are however also other contributions so those quoted above are the only well defined ones. For the $\rho - \omega$ mixing, we obtain $\Delta\rho\omega = 1.8$ MeV.

7 Numerical Results

We take the following inputs: $m_s = 175$ MeV, $m_d = 8$ MeV, $m_u = 4$ MeV, $\alpha_S = 0.3$, $B_0 = m_{K^+}^2/(m_s + m_u) = 1360$ MeV, $F_0 = 93$ MeV, and $m_V = (m_\rho, m_{K^*}, m_\phi)$. For f_V we take 0.2 MeV when $m_V = m_\rho$ and 0.16 MeV when $m_V = m_\phi$. These values follow from (21). When $m_V = m_{K^*}$ we take the intermediate value $f_V = 0.18$ MeV. It follows that $\delta_1 = (2.8, 2.7, 2.4)$ MeV. From the $K^{*+} - \rho^+$ mass difference and (9), we get $a = 1.3 \times 10^{-4}$ MeV $^{-1}$. The parameter d can be obtained from the $\rho - \omega$ mass difference. We have already seen that the ρ^0 , ω and ϕ get mixed. The ϕ mixing can be ignored because the mass splitting $m_\phi - m_{\omega,\rho}$ is much bigger than its mixing and we only need to take $\rho\omega$ mixing into account. Because of the large ρ -width this contribution is also very small. This leads to $d = (6.6, 6.5, 6.5)$ MeV. Next, the $\phi - K^{*0}$ mass difference implies $b = (-2.7, -2.7, -2.6) \times 10^{-6}$ MeV $^{-1}$, which is negligibly small and will only be taken into account when multiplied by m_s . We can attempt to obtain δ_2 by combining ΔK^{*+} , ΔK^{*0} , and $\Delta\rho\omega$, finding $\delta_2 = (1, 5, 1.6, 1.6)$ MeV. At this point it is interesting to make a few checks of our predictions. For the $\rho - \phi$ mixing, (9) predicts $\Delta\rho\phi = (-0.67, -0.63, -0.54)$ MeV, in acceptable agreement with (13). Notice that we also predict the sign, not given in (13). From (10) we get $\Delta\omega\phi = 5.4$ MeV, also in acceptable agreement with (13), but not with (14) We can also make the prediction $m_{\rho^+} - m_{\rho^0} = (-3.0, -2.9, -2.8)$ MeV. It is very difficult to extract δ_3 .

We could try to obtain Λ following [6], were the short and long distance contributions were matched. However, here neither for δ_2 nor for δ_3 is there a matching between the long and short distance contributions. For δ_3 , there are no long distance contributions, while for δ_2 there is a disagreement even in sign. Long and short distance contributions do match for $\Delta\rho^+$ and ΔK^{*+} . We find the rather small values $\Lambda = (233, 218, 205)$ MeV, which imply $\delta_2 = (0, 0.3, 0.6)$ MeV and $\delta_3 = (1.5, 1.9, 2.3)$ MeV.

An acceptable value for Λ should lie somewhere between 500 and 800 MeV, with $-1.8 < \delta_2 < -1.0$ MeV for all three values of m_V , and $(0.13, 0.14, 0.15) < \delta_3 < (0.32, 0.36, 0.39)$ MeV. Using these values we obtain for the mass differences, $\delta V = m_{V^+} - m_{V^0}$ (MeV):

$$(-0.4, -0.3, -0.2) < \delta^{EM} m_\rho < (0.4, 0.5, 0.6) \quad \text{and} \quad 1.1 < \delta^{EM} K^* < 1.8. \quad (35)$$

Here $\delta^{EM} K^*$ is for all three values of m_V . It is very difficult to compare these numbers with the experiment, since for the ρ the present uncertainties are rather large, and the K^* receives important contributions from $m_u - m_d$.

Next, we extract the quark mass ratio

$$-0.026 < \frac{m_u - m_d}{m_s - m_d} = \frac{\Delta\omega\rho - \delta_1/6 - \delta_3/3}{m_{K^{*+}} - m_{\rho^+}} < -0.024, \quad (36)$$

in excellent agreement with [18], who finds -0.025 . The δ_3 contribution is small. Using (35) we also obtain

$$-0.046 < \frac{m_u - m_d}{m_s - m_d} = \frac{m_{K^{*+}} - m_K^{*0} + \delta_2 - \delta_3/3}{m_{K^{*+}} - m_{\rho^+}} < -0.052. \quad (37)$$

These two independent estimates are in disagreement. If we use the value we obtained for δ_3 but the value for δ_2 obtained from the phenomenological estimate $\delta_2 = 1.6 \text{ MeV}$ the second estimate becomes about -0.024 in good agreement with the other one.

The two main effects of $SU(3)$ breaking in the mass differences are the meson loop contributions analyzed in Section 6 and those in the photon-vector meson mixings. Both these effects we can estimate easily. The meson loop results are listed in (33). The other effect on the mass shifts we have already estimated above. For the mixing the situation is more complicated. Here we only need to consider $\Delta\rho\phi$ and $\Delta\omega\phi$. This effect is not important for $\Delta\omega\phi$, dominated by $\sqrt{2}d$ and with δ_1 suppressed. However, a momentum independent $\Delta\rho\phi$ does not make much sense.

The effects from the meson loops are rather large but not quite sufficient to bring the estimate of (37) in line with the standard estimates. The expected shift for the ρ and ω are smaller.

8 Conclusions

We have studied corrections to vector meson masses and mixings. When the strange quark mass is present, it dominates these corrections, the electromagnetic ones are quite small. We neglected corrections of order $O(m_s^2)$. Our numerical predictions agree qualitatively with the observations.

We find that even at lowest order there is no equivalent to Dashen's theorem for vector mesons, not even in the chiral limit due to the presence of the δ_3 term which just follows from group theory.

For $m_q = 0$ we find $\delta^{EM}\rho^+ = \delta^{EM}K^{*+}$, as expected, but $\delta^{EM}\rho^0 \neq \delta^{EM}K^{*0}$, which has two sources: vector-photon mixing, parametrized by δ_1 , and short distance photons, parametrized by δ_3 .

We also made the prediction $-0.4 < \delta^{EM}m_\rho < 0.4 \text{ MeV}$. Here both, the ρ^+ and the ρ^0 receive positive mass corrections, the ρ^+ from (long distance) photons, and the ρ^0 from the mixing with the photon. Taking the estimate of the (34) as

an upper limit on the quark mass corrections, this leads to the final prediction for the $\rho^+ - \rho^0$ mass difference:

$$-0.7 \text{ MeV} < m_{\rho^+} - m_{\rho^0} < 0.4 \text{ MeV}. \quad (38)$$

For the K^* we found using a similar estimate

$$0.5 \text{ MeV} < \delta^{EM} K^* < 1.8 \text{ MeV}. \quad (39)$$

We finally made a prediction for a quark mass ratio in (36) and (37), finding reasonable agreement with existing results, [18] but there are indications that there are significant uncertainties in the estimates of electromagnetic corrections to the vector meson masses.

9 Acknowledgments

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A Cut-off “heavy quark” integrals

The problem is that naively in dimensional regularization integrals like

$$I = \int \frac{d^d p}{(2\pi)^d} \frac{1}{v \cdot p + i\epsilon} \quad (40)$$

vanish. Evaluating this type of integrals with a cut-off is somewhat more tricky. We first rotate to euclidean space and obtain

$$I = i \int^\Lambda \frac{d^4 p_E}{(2\pi)^4} \frac{1}{i v \cdot p_E + i\epsilon}. \quad (41)$$

here we now have to use $1/(x + i\epsilon) = P(1/x) - i\pi\delta(x)$. The principal part of the integral vanishes since it is odd in k_E^0 and the integration regime is symmetric. Here we specialized to $v = (1, 0, 0, 0)$. The remainder spatial three dimensional integral can be easily performed and we obtain

$$I = -\frac{i}{12\pi^2} \Lambda^3. \quad (42)$$

We obtain the same result by taking the integral

$$I_2 = \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{(p + q)^2 - m^2}$$

$$= -\frac{i}{16\pi^2} \int_0^\Lambda \frac{q dq}{p_E^2} \left(q^2 + p_E^2 + m^2 - \sqrt{(q^2 + p_E^2 + m^2)^2 - 4p_E^2 q^2} \right) \quad (43)$$

$$= \frac{-i}{32\pi^2 p_E^2} \left\{ \frac{1}{2} \Lambda^4 + (p_E^2 + m^2) \Lambda^2 - \frac{1}{2} (\Lambda^2 + m^2 - p_E^2) \sqrt{(\Lambda^2 + m^2 - p_E^2)^2 + 4m^2 p_E^2} \right. \\ \left. - 2m^2 p_E^2 \log \frac{\Lambda^2 + m^2 - p_E^2 + \sqrt{(\Lambda^2 + m^2 - p_E^2)^2 + 4m^2 p_E^2}}{m^2 - p_E^2 + |m^2 + p_E^2|} + \frac{1}{2} (m^2 - p_E^2) |m^2 + p_E^2| \right\} \quad (44)$$

going on shell ($p_E^2 = -m^2$), and taking the limit Λ small. Here p_E is the momentum in euclidean space.

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B Figures

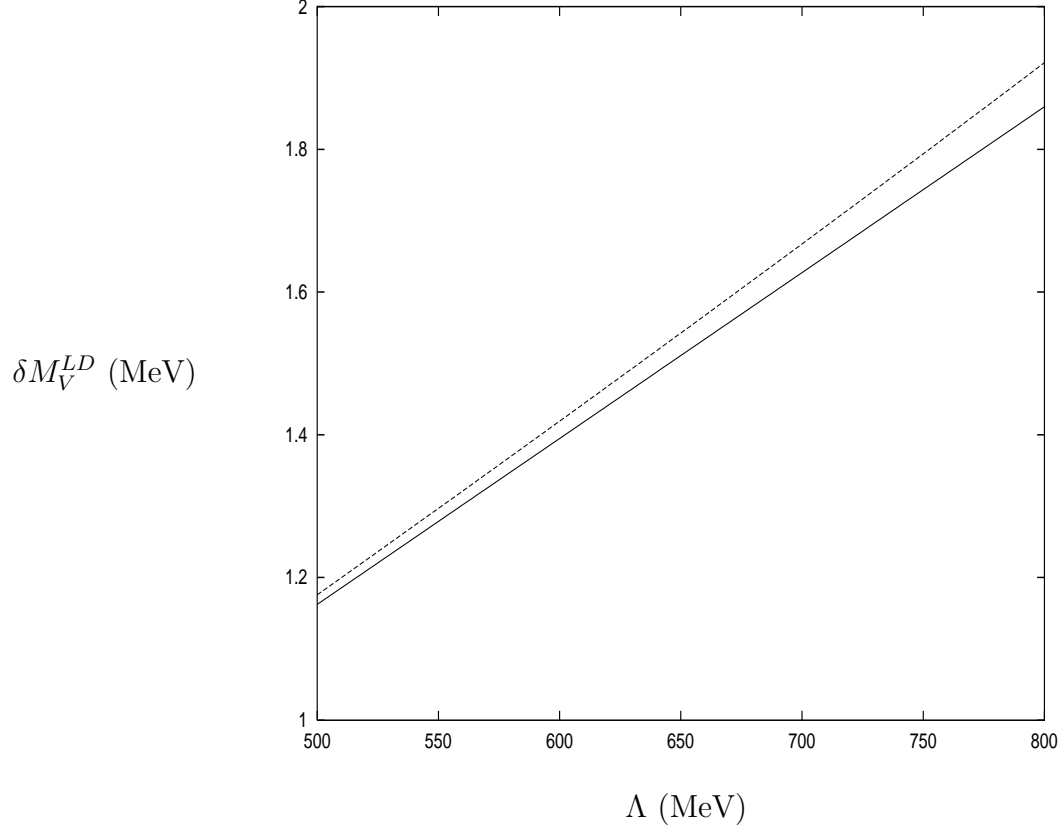


Figure 1: Long distance photon correction to the charged meson masses as a function of the cut-off. The dashed line represents the full relativistic correction, while the straight line represents the heavy meson effective theory prediction.

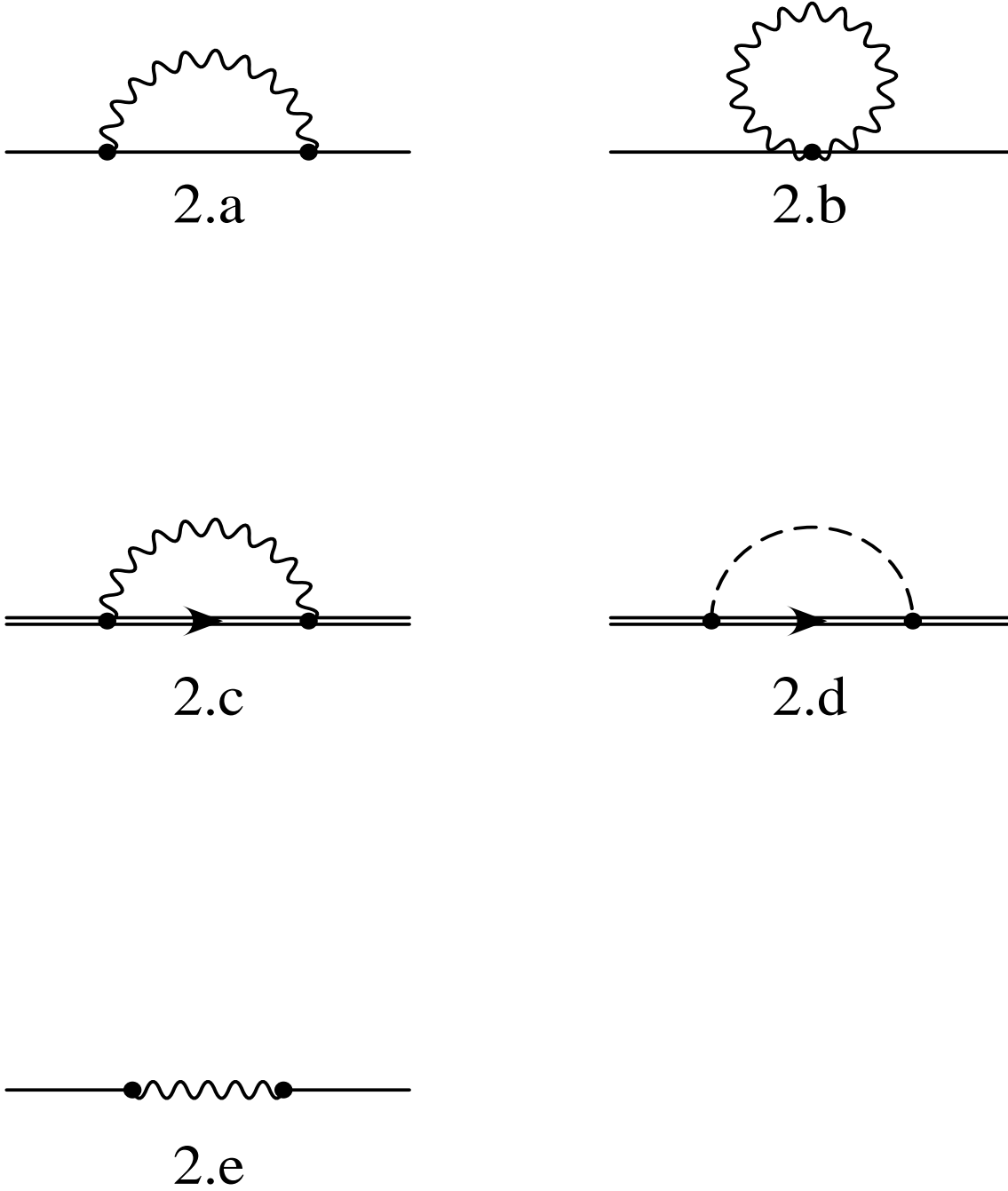


Figure 2: Feynman diagrams contributing to vector meson mass shifts and mixings. The straight lines in 2.a and 2.b denote the mesons and the curly line the photon. 2.b does not contribute when we using dimensional regularization. The double lines in 2.c and 2.d denote heavy mesons. They carry an arrow because their field destroy particles, but do not create antiparticles. Here a diagram like 2.b is suppressed by $1/m_V$. The dashed line in 2.d represents a pseudoscalar meson. The meson-photon mixing contribution is represented in 2.e.